Asymptotic Analysis of Stagnating Turbulent Flows

Michel Champion*
ENSMA, Poitiers 86034, France
and
Paul A. Libby†
University of California, San Diego, California 92093

The k-ε theory is applied to describe turbulence stagnating against a wall in the limit of large Reynolds numbers. Such turbulence involves three regions: a viscous sublayer, a shear layer, and the external stream approaching the wall. Since each region involves distinct velocity and length scales, an asymptotic analysis is indicated but the requirements of matching solutions for adjacent regions are found to necessitate adjustments of the empirical constants in the dissipation equation. Two examples of approaching streams are considered: decaying grid turbulence approaching a bluff cylindrical body and a turbulent slot flow impinging on a wall. The analysis shows that the viscous sublayer differs significantly from its counterpart in turbulent boundary layers in several respects; in particular, it is found to account for the entire change in streamwise velocity and mean temperature across the shear layer and to be relatively thick.

I. Introduction

RECENT papers, such as Strahle et al.[1] on stagnating turbulent flows, Bray et al.,[2] the first of a series of papers on premixed flames in such flows, and Cho et al.,[3] which presents experimental data for such flames, indicate current interest in the normal impingement of a turbulent flow against a wall. The fact that these contributions continue a long series of studies of this flow, principally motivated by concern for the influence of freestream turbulence on the heat transfer at stagnation points, is indicated by the extensive list of references in Strahle et al.[1]

Stagnating turbulent flows consist of three regions with distinct length scales: a viscous sublayer; a shear layer involving the full range of turbulent processes, convection, turbulent diffusion, production and dissipation; and an outer flow involving decaying turbulence, which is strained as the stagnation plane is approached. We denote these as regions I–III, respectively. Of particular interest are the differences between the usual viscous sublayer of a turbulent boundary layer and the flow in region I, differences due to the distinct mean rates of strain that prevail in the two cases. Since the three regions have widely different characteristic length scales, their description by an asymptotic analysis is clearly indicated. The purpose of this paper is to provide such an analysis for a stagnating flow with small degrees of heat transfer.

Our treatment of regions I and II follows the early work of Bush and Fendell[4] on the asymptotic analysis of turbulent channel and boundary-layer flows. Thus, a set of equations applicable from the wall to the turbulent external flow, i.e., to the inner edge of region III, is considered. As in Strahle et al.,[1] we employ for this set the k-ε model of turbulence with low Reynolds number effects taken into account. These equations apply to the relatively thin viscous sublayer involving both molecular and turbulent transport and to region II where the latter transport dominates. This division of the flow leads naturally to separate descriptions of the two regions and to an asymptotic analysis. Matching provides the boundary conditions to be applied "at the wall" for the equations applicable to region II (equations corresponding to the standard k-ε model) in which molecular transport is absent.

Whether an asymptotic analysis is applied to the description of regions I and II or whether, as in Strahle et al.[1] and Traci and Wilcox,[7] unified solutions for the two regions are obtained for a specified Reynolds number, we assume that regions I and II are so thin compared with region III that it is necessary to call for boundary conditions to be imposed at "infinity" on the dependent variables relevant to region II. This assumption implies that, at the outer edge of region II, the production and dissipation of turbulent kinetic energy are balanced, an assumption also employed by Strahle et al.[1] that calls for modification of the empirical constants in the dissipation equation.

The analysis of regions I and II applies to various cases of stagnating turbulence, e.g., grid turbulence approaching a bluff cylindrical body and turbulence from a two dimensional slot impinging on a wall. Although the former application corresponds to the analysis of Strahle et al.[1] and Traci and Wilcox[7] and to studies of the influence of freestream turbulence on the heat transfer to stagnation points, to reinforce the notion that the analysis of regions I and II applies to various cases of stagnating turbulence we work out the details of the two applications cited earlier, identifying them as grid and slot turbulence, respectively.

When the realistic assumption of relatively weak turbulence is applied to region III, it is found by analysis that for both grid and slot turbulence the mean velocity components are given by potential flow considerations, a result reminiscent of the application of rapid distortion theory to grid turbulence approaching a cylindrical body by Britter et al.[5] and Bearman[6] and of the approximation employed by Traci and Wilcox[7] in their region 2, which is an idealization of our region III. Thus, the equations describing region III and leading to a consistent calculation are those for the turbulent kinetic energy and viscous dissipation with a given mean velocity field and, as a consequence of the applicable scaling, with negligible turbulent diffusion. The solution of these equations from specified initial values at the outer edge of region II describes the changes with distance from the stagnation plane of the k and ε. For grid turbulence, we consider the potential flow of velocity V_0 about a circular cylinder of radius R, whereas for slot turbulence we consider a uniform stream of velocity V_0 exiting from a slot located a distance d from an infinite wall. The treatment of regions I and II is

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*Director of Research, Laboratoire d'Energetique, URA-CNRS 183.
†Professor of Fluid Mechanics. Fellow AIAA.

general, being applicable to any stagnating turbulent flow resulting in a mean rate of strain a and a turbulent kinetic energy k_∞ at the outer edge of region II.

Our analysis differs from that of Strahle et al.[1] and Traci and Wilcox[7] in several respects. In the previous work, the describing equations in regions I and II are solved directly by numerical means, the former by the k-ε method with Favre averaging, and the latter by the Saffman turbulence model with constant fluid properties. These solutions are identified with particular values of one or more Reynolds numbers, of the turbulent kinetic energy at the outer edge of region II and, in the case of Strahle et al.,[1] the ratio of wall to external stream temperatures. Here, we employ asymptotic methods and describe the two regions separately. Since we consider only the lowest order solutions, no Reynolds number need be specified. Our analysis also differs in some detail. For example, our basic similarity variable, that applicable to region II, is defined in terms of the two parameters cited earlier and identifiable in applications to various cases of region III rather than in terms of a representative laminar coefficient. Thus, our equations only resemble those of Strahle et al.[1] The independent variables for regions I and III are obtained by appropriate scaling of this basic similarity variable. Also, we include explicit calculations of the flow in region III for grid and slot turbulence. Finally, our treatment of the two empirical constants in the dissipation equation is somewhat different. In addition to the modification noted earlier arising from the requirements of matching regions II and III, we find that matching of regions I and II calls for a further modification of these constants. In this regard, it is remarked that the need for this second modification may account for the numerical difficulties in the direct solution of the k-ε equations reported by Strahle et al.,[1] difficulties that increase with increasing Reynolds number but which, of course, they overcome.

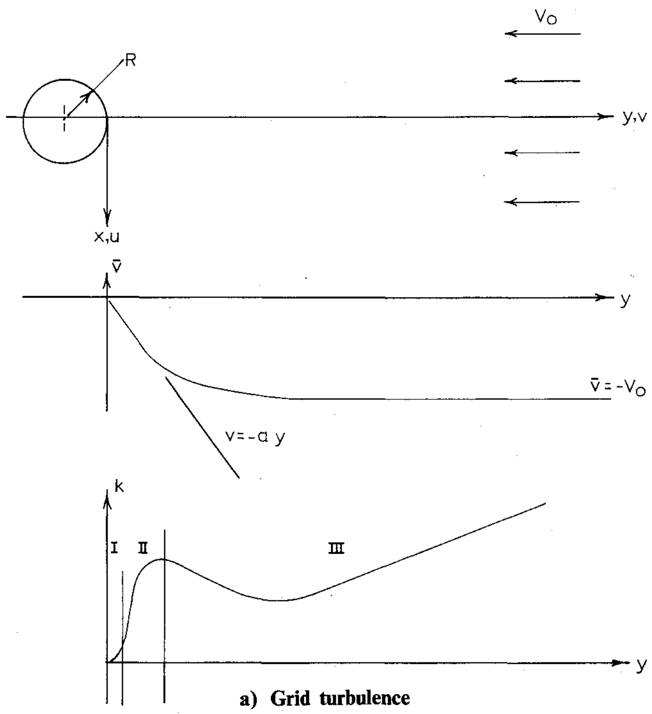
After the present study was underway, the contribution of Taulbee and Tran[8] appeared. These authors point out that the k-ϵ theory in its usual form does not accurately predict the flow in region III. However, in a responding technical note[9] the present authors show that if the empirical constants in the dissipation equation are varied so as to reflect the matching requirements of regions II and III, the k-ε theory can be rehabilitated for this application. It is also noted in Ref. 9 that the need to adjust the empirical constants in the k-ε calculation reflects a shortcoming of the theory, one presumably remedied by use of a Reynolds stress closure as pointed out by Taulbee and Tran.[8] Our finding of the need for a further modification of the constants in the dissipation equation to match regions I and II likewise suggests a possible additional shortcoming of the theory for these stagnating flows. The discussion in Ref. 9 is an abbreviated version of the treatment of region III for grid turbulence given here.

The contribution of Taulbee and Tran[8] calls attention to the work of Hijikata et al.,[10] a combined theoretical and experimental investigation of a circular cylinder crossflow to a turbulent stream. The theory involves supplementing the k-ε model with an anisotropy equation accounting for the difference in the intensities of two orthogonal velocity components. Measurements of the mean velocity and intensity of the velocity fluctuations along the dividing streamline are reported (measurements used in Refs. 8 and 9). The predictions of the theory relative to the influence of freestream turbulence on the heat transfer to the cylinder are compared with the earlier data of Smith and Kuethe.[11] However, there appear to be no detailed turbulence measurements applicable to regions I and II and suitable for comparison with the present theory.

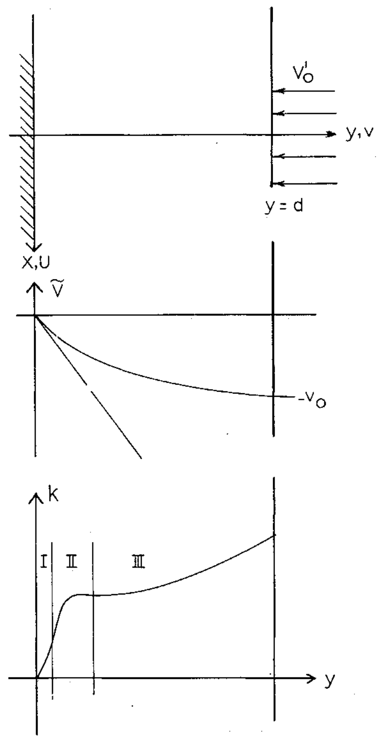
Our focus is on the velocity characteristics of stagnating turbulence but since the influence of freestream turbulence on stagnation point heat transfer provides the motivation for the related studies of stagnating turbulence, we treat the mean temperature equation for rates of heat transfer sufficiently low so that the fluid properties may be considered constant. Thus, the case of heated or cooled walls, either of bluff bodies in grid turbulence or of surfaces exposed to slot turbulence, are analyzed.

II. Analysis

Figure 1 shows schematically the flow and distributions of two key flow variables for the two cases considered here. Figure 1a relates to a uniform, weak turbulent flow encountering a bluff body such as a circular cylinder. Thus, the mean velocity component $\bar{v}$ along the plane of symmetry changes from a constant value $-V_0$ to linearly decreasing values as the stagnation plane is approached. The linear distribution is described by a mean rate of strain parameter a, which plays a



a) Grid turbulence



b) Slot turbulence

Fig. 1 Schematic representation of stagnating turbulent flow.

central role in scaling the variables in all three regions. Since we consider the flow to correspond to that about a circular cylinder, $a = 2V_0/R$ where R is the radius of the cylinder, but the analysis of regions I and II applies to any flow provided $\bar{v} \propto -ay$ in the outer portions of region II. Finally, the distribution of turbulent kinetic energy is shown schematically; in contrast with the usual turbulent boundary layer the present flow involves a monotonic increase in that energy from zero at the wall to a relatively constant value at the outer edge of region II, a decrease in the inner portion of region III, and, finally, an indefinite linear increase with distance from the stagnation point. The rates of change of the turbulent kinetic energy are significantly different in the three regions; indeed, the initial rate of decrease in region III is so small that matching of the solutions in regions II and III calls for a constant value of that energy as $\eta \to \infty$, a value denoted k_∞ where η is the basic similarity variable mentioned earlier. Along with the rate of strain parameter a, the quantity k_∞ plays a central role in the analysis. The relation between k_∞ and the nature and location of a turbulence generating grid in a particular application of the present analysis is defined by region III, i.e., the turbulence in region III acccommodates the turbulent kinetic energy at the edge of the shear layer. Although not shown in Fig. 1a, we would expect that the mean temperature $\bar{T}$ varies monotonically from a wall value denoted T_w to T_∞ at the outer edge of region II and remains at that value throughout region III with the wall temperature either greater or less than T_∞. However, we show later that for suitably large Reynolds numbers essentially the entire change in the mean temperature occurs within the viscous sublayer. The viscous dissipation is nonzero at the wall and increases in regions I and II and then decreases in the inner portions of region III before monotonically increasing at large distances from the body. The significant differences between the behavior of variables in the shear layer of a stagnating flow and the usual turbulent boundary layer should be kept in mind during the subsequent discussion.

Figure 1b shows schematically the second flow considered here. The turbulent flow with a mean velocity V_0 issues from a slot located d from an infinite wall. Again we have a decay of the turbulence until the rate of strain associated with the wall becomes effective: a thin shear layer adjacent to the wall and an even thinner viscous sublayer immediately contiguous with the wall. We shall see later that the rate of strain at the edge of the shear layer is in this case $\pi V_0/2d$. The behavior of the turbulent kinetic energy, viscous dissipation, and the mean temperature in regions I and II, in this case, is the same as in grid turbulence.

In both grid turbulence and slot flow, the length scale of the freestream turbulence is assumed to be small compared with the radius of the cylinder R and the slot spacing d, respectively. This is fundamentally necessary for a gradient transport model, e.g., the k-ε model, to apply.

As indicated earlier, the asymptotic analysis of this flow involves first consideration of the equations applicable to both regions I and II. Arising in these equations is a small parameter identified with molecular transport and, thus, with an inner independent variable $\hat{\eta}$ appropriate for the description of region I. When this small parameter is set to zero and several empirical functions are set to unity, there result equations for the lowest order description of the flow in region II. The analysis for two different examples of region III, grid and slot turbulence as discussed earlier, is handled subsequently.

Preliminary Considerations for Regions I and II

We introduce the similarity variable

$$\eta \equiv (a/k_\infty^{1/2})y \tag{1}$$

It is worth noting that the only length scale arising in these flows is $k_\infty^{1/2}/a$, i.e., no turbulence length scale enters explicitly in the analysis. However, if the scale of the turbulence approaching the shear layer is changed, then k_∞ and, thus, the thickness of that layer is altered so that there is an implicit dependence on turbulent length scales. The characterization of the flow in regions I and II by the two parameters k_∞ and a permits various examples of region III to be considered.

The mean velocity components are determined in terms of a modified stream function $f(\eta)$ such that

$$\bar{u} = axf'(\eta) \tag{2a}$$

$$\bar{v} = -k_\infty^{1/2}f(\eta) \tag{2b}$$

The other dependent variables are the mean temperature $\bar{T}(\eta)$, the turbulent kinetic energy $k(\eta)$, and the viscous dissipation $\varepsilon(\eta)$. These are made dimensionless according to

$$\bar{T} = T_w + (T_\infty - T_w)g(\eta) \qquad k = k_\infty K(\eta) \qquad \varepsilon = ak_\infty E(\eta) \tag{3}$$

In the averaged conservation equations, there appear a variety of second moment quantities whose functional dependence on the x coordinate is dictated by symmetry and antisymmetry considerations. Thus, we introduce the following definitions:

$$\overline{u'^2} = k_\infty G_{uu}(\eta), \quad \overline{v'^2} = k_\infty G_{vv}(\eta), \quad \overline{w'^2} = k_\infty G_{ww}(\eta)$$
$$\overline{u'v'} = ak_\infty^{1/2}xG_{uv}(\eta), \quad \overline{u'T'} = axT_\infty G_{uT}(\eta)$$
$$\overline{v'T'} = k_\infty^{1/2}T_\infty G_{vT}(\eta) \tag{4}$$

Later we eliminate the G functions by means of a gradient transport assumption. Finally, a form for the mean pressure, quadratic in the x coordinate with an additive arbitrary function of η is also assumed [cf. Eq. (29)] but this function plays no essential role in the analysis and need not be explicitly included.

We now develop the similarity forms of the conservation equations needed to describe regions I and II, namely those for x-wise momentum, temperature, turbulent kinetic energy and viscous dissipation. Exposition is facilitated if, as an intermediate step, we write these equations in terms of the G functions of Eqs. (4) and subsequently introduce gradient transport.

Equations for Regions I and II

The following are the equations for x-wise momentum and mean temperature conservation:

$$\{-G_{uv} + [(va)/(k_\infty)]f''\}' + ff'' + 1 - f'^2 = 0 \tag{5}$$

$$\{-G_{vT} + (va)/(k_\infty)](g'/\sigma)\}' + fg' - G_{uT} = 0 \tag{6}$$

where σ is the molecular Prandtl number.

In Eqs. (5) and (6) we now incorporate gradient models for the turbulent stresses and fluxes. Accordingly, we let

$$-\overline{u_i'u_j'} = v_T\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right) - \frac{2}{3}k\delta_{ij} \tag{7a}$$

$$-\overline{u_i'T'} = \frac{v_T}{\sigma_T}\frac{\partial \bar{T}}{\partial x_i} \tag{7b}$$

where σ_T is another empirical constant: the turbulent Prandtl number.

Application of Eqs. (7) to the definitions of Eqs. (4) leads to the following expressions for the G functions:

$$G_{uu}(\eta) = (2/3)[K - 3(v_Ta/k_\infty)f'] \tag{8a}$$

$$G_{vv}(\eta) = (2/3)[K + 3(v_Ta/k_\infty)f'] \tag{8b}$$

$$G_{ww}(\eta) = (2/3)K \tag{8c}$$

$$G_{uv}(\eta) = -(v_T a/k_\infty)f'' \tag{8d}$$

$$G_{uT}(\eta) = 0 \tag{8e}$$

$$G_{vT}(\eta) = -(v_T a/k_\infty)(g'/\sigma_T) \tag{8f}$$

Although the dimensionless intensity of the w-velocity component is not needed, it is included here for completeness. According to the k-ε theory, the exchange coefficient is given by

$$v_T = c_\mu f_\mu (k^2/\varepsilon)$$

where f_μ is a wall function accounting for low Reynolds number effects that are discussed later. Thus, the dimensionless turbulent exchange parameter in Eqs. (8) is

$$(v_T a/k_\infty) = c_\mu f_\mu (K^2/E) \tag{9}$$

Before setting forth the final two equations (those for the turbulent kinetic energy and the viscous dissipation) some discussion is indicated. There are various modifications of the k-ε equations to incorporate the effect of low turbulence Reynolds numbers. Patel et al.[12] provide a useful, recent survey and assessment of these modifications including comparison of the predictions of the various equations with a variety of experimental data. One of the superior sets of equations is due to Lam and Bremhorst[13] (LB), and since this is adopted by Strahle et al.,[1] we do likewise. The principal modifications related to low Reynolds number effects are in the viscous dissipation equation; thus, the LB model involves three wall functions that become unity at the inner edge of region II and thereby reduce the equations to standard k-ε form. We accept the approximation of Strahle et al.[1] and neglect the contribution of the normal pressure gradient $\partial \bar{p}/\partial y$ to the balance of the turbulent kinetic energy and viscous dissipation.

Thus, we arrive at the following additional equations:

$$\{[(v_T a/\sigma_k k_\infty) + (va/k_\infty)]K'\}' + fK' + P - E = 0 \tag{10a}$$

$$\{[(v_T a/\sigma_\varepsilon k_\infty) + (va/k_\infty)]E'\} + fE' + (E/K)(c_{\varepsilon 1} f_1 P - c_{\varepsilon 2} f_2 E) = 0 \tag{10b}$$

where

$$P = -f'(G_{uu} - G_{vv}) = 4c_\mu f_\mu (K^2/E) f'^2$$

describes the production of turbulence. It is worth noting in these flows that turbulent production is associated with the normal stresses in contrast with the more common shear flow applications in which the shear stresses lead to such production. In Eq. (9) and Eqs. (10), there appear three wall functions, namely,

$$f_1 = 1 + (A_1/f_\mu)^3, \qquad f_2 = 1 - \exp\{-[(k_\infty/va)(K^2/E)]^2\} \tag{11a}$$

$$f_\mu = \left[1 - \exp\left(-A_\mu K^{1/2}\frac{k_\infty}{va}\eta\right)\right]^2 \left[1 + \frac{A_t}{(k_\infty/va)(K^2/E)}\right] \tag{11b}$$

where A_1, A_μ, and A_t are empirical constants given by the LB theory.

The formulation applicable to regions I and II is now complete; Eqs. (5), (6), and (10) can be solved for f, K, and E and, subsequently, for g subject to appropriate boundary conditions at $\eta = 0$ and at the outer edge of region II, namely to:

At $\eta = 0$:

$$f = f' = K = 0, \qquad E = E_w, \qquad g = 0 \tag{12}$$

At $\eta \to \infty$:

$$f' = g = K = 1, \qquad E = E_\infty = 2c_\mu^{1/2} \tag{13}$$

The value of E_w is obtained from the analysis as discussed in detail later.

Some comments regarding the conditions at $\eta \to \infty$ are indicated. Implicit in the imposition of conditions at $\eta \to \infty$ is the assumption that, as mentioned earlier, regions I and II are thin compared with region III. Within the context of an asymptotic analysis, this assumption calls for matching the solutions for regions II and III, i.e., holding fixed the independent variable for the latter region $\tilde{\eta}$ introduced later at a small nonzero value and allowing an expansion parameter $\tilde{\delta}$, also introduced later, to go to zero. The result is that boundary conditions for region II are imposed at $\eta \to \infty$. Furthermore, again as suggested earlier, Eqs. (5) and (6) are consistent with imposition of conditions at $\eta \to \infty$ only if $P - E \to 0$ with the consequence that the two separate empirical constants, $c_{\varepsilon 1}$ and $c_{\varepsilon 2}$, must be equal as $\eta \to \infty$ and that E must take on the indicated value. This requirement is first noted by Strahle[14] and used in Strahle et al.[1] and Bray et al.[2] In the former reference, it is argued that the usual value for $c_{\varepsilon 1}$ should be increased so as to equal $c_{\varepsilon 2}$ throughout regions I and II, whereas the present authors[9] find that if $c_{\varepsilon 1} = c_{\varepsilon 2} = c_\varepsilon = 2.27$, good agreement with experimental results in region III is obtained. Accordingly, we require that this value prevail at the outer edge of region II, but as noted earlier we find that the matching of regions I and II calls for a distribution of these constants within region II.

Asymptotic Treatment of Regions I and II

In addition to specification of the various empirical constants, numerical solutions of these equations require selection of the parameter va/k_∞, which is the reciprocal of a Reynolds number based on k_∞ and the rate of strain parameter a. Except for the modifications introduced in the present analysis, such solutions are obtained by Strahle et al.[1] Here, we take advantage of the small size of the values of $\tilde{\delta} \equiv va/k_\infty$ of applied interest and develop an asymptotic analysis, i.e., we let $\tilde{\delta} \to 0$, a limit that implies that the laminar exchange coefficient becomes vanishingly small and that a new independent variable is required to resolve region I.

It is worth examining the parameter $\tilde{\delta}$ further for both grid and slot turbulence. For the former case, we consider the bluff body to be a circular cylinder of radius R so we have

$$\tilde{\delta} = \frac{v}{V_0 R}\frac{2}{k_\infty/V_0^2}$$

Thus, we see that $\tilde{\delta}$ is small if the Reynolds number of the body is large and the turbulence intensity is large. The former requirement is likely to be satisfied in experiments, but in grid turbulence the relative intensity $k_\infty^{1/2}/V_0$ tends to be small so that in some past experiments in grid turbulence our expansion parameter may not be sufficiently small for the present analysis to apply. To examine this point further, let $V_0 R/v = 10^4$; if $k_\infty^{1/2}/V_0 = 10^{-2}$, then $\tilde{\delta}$ is unity, not suitably small. However, if the turbulent intensity is suitably high, i.e., if $k_\infty^{1/2}/V_0 = 10^{-1}$, then $\tilde{\delta} = 10^{-2}$ and we can expect the present analysis to apply.

For slot turbulence we have

$$\tilde{\delta} = \frac{\pi}{2}\frac{v}{V_0 d}\frac{1}{k_\infty/V_0^2}$$

which closely resembles the result for grid turbulence. Indeed, the previous considerations for the need for both large Reynolds numbers and large relative intensities for δ to be suitably small apply.

There are several requirements for scaling the independent and dependent variables applicable to region I; we let $\hat{\eta}$ denote the former and $\hat{f}$, $\hat{K}$, $\hat{E}$, and subsequently $\hat{g}$ the latter variables. The requirements on these variables in region I are

1) $\hat{K}^2/\hat{E}$ must be on the order of δ.

2) Convective effects must be negligible compared with diffusive effects.

3) The independent variable $\hat{\eta}$ must be stretched so that matching involves $\hat{\eta} \to \infty$ as $\eta \to 0$. On the basis of these considerations we let

$$\hat{\eta} \equiv \eta/\delta^{2/3}, \qquad \hat{f} \equiv f/\delta^{2/3}, \qquad \hat{g} \equiv g \tag{14a}$$

$$\hat{K} \equiv K/\delta^{2/3}, \qquad \hat{E} \equiv E/\delta^{2/3} \tag{14b}$$

Given the absence of a shearing velocity in stagnating turbulent flows, it should not be surprising that the independent variable for the viscous sublayer is not the usual y^+. In this regard, it is illuminating to consider $\hat{\eta}$ in terms of primitive variables: we have

$$\hat{\eta} = \frac{k_\infty^{1/6} a^{1/3}}{v^{2/3}} y$$

We see that the thickness of region I depends, in a complex fashion, on various flow variables but its dependence on those variables is physically appealing. For example, if the mean rate of strain increases, we expect and indeed find that the sublayer thickness decreases. Note the weak dependence of the viscous sublayer thickness on k_∞; this $k_\infty^{1/6}$ dependence appears repeatedly in the present analysis and is a direct consequence of the scaling requirements associated with large Reynolds numbers.

In a complete asymptotic analysis, all dependent variables in region I are expanded in terms of δ, i.e.,

$$\hat{f}(\hat{\eta};\delta) = \hat{f}_0(\hat{\eta}) + \delta^{1/3}\hat{f}_1(\hat{\eta}) + \cdots$$

with similar expansions, i.e., in terms of $\delta^{1/3}$, for the other dependent variables. However, since we confine our attention to the lowest order terms, we avoid the unnecessary clutter of subscripts on the dependent variables with the understanding that they implicitly corresond to zero order in δ.

Equations for Region I

When the new variables are introduced into Eq. (5) and Eqs. (10) and when the terms independent of δ are collected, the following equations describing the velocity characteristics are obtained:

$$\left(1 + c_\mu f_\mu \frac{\hat{K}^2}{\hat{E}}\right)\hat{f}'' = \hat{f}''(0) \tag{15a}$$

$$\left[\left(1 + \frac{c_\mu f_\mu}{\sigma_k}\frac{\hat{K}^2}{\hat{E}}\right)\hat{K}'\right]' - \hat{E} = 0 \tag{15b}$$

$$\left[\left(1 + \frac{c_\mu f_\mu}{\sigma_\varepsilon}\frac{\hat{K}^2}{\hat{E}}\right)\hat{E}'\right]' - c_{\varepsilon 2} f_2 \frac{\hat{E}^2}{\hat{K}} = 0 \tag{15c}$$

where prime now denotes differentiation with respect to $\hat{\eta}$. It is interesting to note that to lowest order in δ there is no turbulence production in either the k or ε equations and that diffusion balances dissipation in these equations in contrast with the usual balance of production and dissipation at the outer edge of the viscous sublayer in turbulent boundary layers. This difference may be expected to result in a significantly different role for the viscous sublayer in the shear layers under consideration than in turbulent boundary layers, although the implication of the first of these equations is that as in the usual viscous sublayer the mean shear stress is constant to lowest order in δ.

The wall functions for Eqs. (15) become

$$f_\mu = [1 - \exp(-A_\mu \hat{K}^{1/2}\hat{\eta})]^2\left(1 + \frac{A_l}{(\hat{K}^2/\hat{E})}\right) \tag{16a}$$

$$f_2 = 1 - \exp\left[-\left(\frac{\hat{K}^2}{\hat{E}}\right)^2\right] \tag{16b}$$

Equations (16) describe the temperature characteristics and for the terms independent of δ yields

$$\left(1 + \frac{\sigma c_\mu f_\mu}{\sigma_T}\frac{\hat{K}^2}{\hat{E}}\right)\hat{g}' = \hat{g}'(0) \tag{17}$$

Here, we see that the heat flux is constant in the viscous sublayer as in the usual turbulent boundary layer.

The boundary conditions of Eqs. (12) clearly apply at $\hat{\eta} = 0$ to Eq. (15) and (17), but we must consider the situation $\hat{\eta} \to \infty$. In this limit, Eqs. (15b) and (15c) imply that $\hat{K} \to \hat{\eta}$ and $\hat{E} \to \hat{\eta}^{1/2}$ and, thus, that $\hat{K}^2/\hat{E} \to \hat{\eta}^{3/2}$. This behavior differs from that assumed in the theoretical analysis of Smith and Kuethe[10] and from that found in turbulent boundary layers, but these differences do not warrant concern given the absence of definitive experimental data to the contrary and the significant differences between stagnation point and boundary-layer flows. Now we know either from previous studies (cf. Strahle et al.[1] and Patel et al.[12]) or from direct consideration of the velocity fluctuations in the neighborhood of the wall that as $\hat{\eta} \to 0$ $\hat{K} = \alpha_K \hat{\eta}^2$ where α_K is arbitrary. From Eq. (15b), we find that as $\hat{\eta} \to 0$ $\hat{E} = 2\alpha_K$ so that $\hat{E}_w$ is determined by α_K. Thus, Eqs. (15b) and (15c) are solved with α_K and $\hat{E}'(0)$ selected so that as $\hat{\eta} \to \infty$ $\hat{K} = \hat{A}_K \hat{\eta}$ and $\hat{E} = \hat{A}_E \hat{\eta}^{1/2}$ where $\hat{A}_K$ and $\hat{A}_E$ are prescribed. We shall see later that these prescribed values are given by matching with the solutions applicable to region II.

With the behavior of $\hat{K}$ and $\hat{E}$ established, Eq. (15a) implies that as $\hat{\eta} \to \infty$ $\hat{f}' \to \hat{f}'_\infty$ where the latter is proportional to $\hat{f}''(0)$, a quantity as yet undetermined. Moreover, from Eq. (17) we see that $\hat{g}$ approaches a constant dependent on $\hat{g}'(0)$, also as yet undetermined.

Equations for Region II

Equations (5), (6), and (10) with the molecular terms and the wall functions set to zero and unity, respectively, yield to lowest order in δ the equations for region II and have the standard k-ε form, i.e., we have

$$c_\mu[(K^2/E)f'']' + ff'' + 1 - f'^2 = 0 \tag{18a}$$

$$(c_\mu/\sigma_T)[(K^2/E)g']' + fg' = 0 \tag{18b}$$

$$(c_\mu/\sigma_k)[(K^2/E)K']' + fK' + P - E = 0 \tag{18c}$$

$$(c_\mu/\sigma_k)[(K^2/E)E']' + fE' + (E/K)(c_{\varepsilon 1}P - c_{\varepsilon 2}E) = 0 \tag{18d}$$

Equations (18a), (18c), and (18d) are solved first and yield the velocity characteristics while Eq. (18b) is solved subsequently for the mean temperature.

According to the strategy discussed earlier, these equations are to be solved subject to the following conditions at $\eta \to \infty$:

$$f' = g = K = 1, \qquad E = E_\infty = 2c_\mu^{1/2} \tag{19}$$

while the boundary conditions at $\eta = 0$ are obtained by matching the solutions for regions I and II. Preliminary considerations suggest the following expansions applicable to the neighborhood of $\eta = 0$:

$$f \approx f'(0)\eta + A_f \eta^{3/2} + \cdots \tag{20a}$$

$$K \approx A_K\eta + \cdots, \qquad E \approx A_E\eta^{1/2} + \cdots \tag{20b}$$

$$g \approx g(0) + A_g\eta^{1/2} \tag{20c}$$

where we expect a priori that A_K and A_E must be selected to satisfy the boundary conditions on $K(\eta)$ and $E(\eta)$ as $\eta \to \infty$. Equations (20) constitute the boundary conditions for the equations for region I to be applied at the wall, $\eta = 0$. Furthermore, if $f'(0)$ and $g(0)$ are assumed known, then A_f and A_g are selected so that the boundary conditions on $f'(\eta)$ and $g(\eta)$ as $\eta \to \infty$ are satisfied.

Matching the solutions for regions I and II to lowest order in $\hat{\delta}$ yields

$$\lim_{\hat{\eta}\to\infty} \hat{K} - A_K\hat{\eta} = 0 \tag{21a}$$

$$\lim_{\hat{\eta}\to\infty} \hat{E} - A_E\hat{\eta}^{1/2} = 0 \tag{21b}$$

and we thus see that $\hat{A}_K = A_K$ and $\hat{A}_E = A_E$ so that the boundary conditions applicable to Eqs. (15b) and (15c) are complete, i.e., α_K and $\hat{E}'(0)$ are selected so that as $\hat{\eta} \to \infty$, $\hat{K} = A_K\hat{\eta}$, and $\hat{E} = A_E\hat{\eta}^{1/2}$ where A_K and A_E are given by the solutions for region II. Similarly, matching the x-wise velocity implies that $f'(0) = \hat{f}'(\hat{\eta} \to \infty)$, but the latter is linearly dependent on the unknown shear stress parameter $\hat{f}''(0)$. Finally, $g(0) = \hat{g}(\hat{\eta} \to \infty)$ with the latter value dependent on the heat transfer parameter $\hat{g}'(0)$.

We now examine the implications of Eqs. (20). Substitution into Eqs. (18) yields the following relations among the coefficients of these expansions:

$$\frac{3}{4}c_\mu \frac{A_f A_K^2}{A_E} = -1 + f'^2(0) \tag{22a}$$

$$\frac{3}{2}\frac{c_\mu}{\sigma_k}\frac{A_K^3}{A_E} = A_E \tag{22b}$$

$$\frac{1}{2}\frac{c\mu}{\sigma_\varepsilon}A_K^3 = c_{\varepsilon 2}A_E^2 \tag{22c}$$

$$A_g = 0 \tag{22d}$$

Equations (22b) and (22c) reflect the balance of turbulent diffusion and dissipation at the inner edge of region II as called for by Eqs. (15b) and (15c), i.e., by the equations for region I.

There are several conclusions to be drawn from Eqs. (22). The most drastic is that the standard values of the empirical constants must be modified so that Eqs. (22b) and (22c) are consistent, i.e.,

$$\frac{3}{\sigma_k} = \frac{1}{\sigma_\varepsilon c_{\varepsilon 2}} \tag{23}$$

If we retain standard values for the empirical constants associated with turbulent diffusion, we must interpret Eq. (23) as determining the value of $c_{\varepsilon 2}$ at $\eta \to 0$, a value we denote $c_{\varepsilon 20}$. Moreover, we shall find that we must allow the second constant in the dissipation equation to vary across region II. Thus, we take

$$c_{\varepsilon 1} = c_{\varepsilon\infty} + (c_{\varepsilon 10} - c_{\varepsilon\infty})\exp(-\eta^2) \tag{24a}$$

$$c_{\varepsilon 2} = c_{\varepsilon\infty} + (c_{\varepsilon 20} - c_{\varepsilon\infty})\exp(-\eta^2) \tag{24b}$$

where $c_{\varepsilon\infty} = 2.27$, $c_{\varepsilon 20}$ is given by Eq. (23), and $c_{\varepsilon 10}$ is as yet undetermined. If Eq. (23) is respected, then from Eqs. (22) the two parameters A_K and A_E are related according to

$$A_E^2 = \frac{3}{2}\frac{c_\mu}{\sigma_k}A_K^3 \tag{25}$$

We later discuss the implications of this finding relative to the numerical analysis of Eqs. (18).

The second implication from Eqs. (22) is that a solution of Eq. (22a) is $f'(0) = 1$, $A_f = 0$, indicating that at high values of the Reynolds number $k_\infty/\nu a$ the entire change in the $\bar{u}$-velocity component across the shear layer occurs within region I and that $\hat{f}''(0)$ must be selected so that $\hat{f}'(\hat{\eta} \to \infty) = 1$. A related solution to Eq. (18b) is $g \equiv 1$, the implication being that $\hat{g}'(0)$ must be selected so that $\hat{g}(\eta \to \infty) = 1$ and, thus, that the entire change in the temperature occurs within the viscous sublayer to lowest order in $\hat{\delta}$. These findings are not inconsistent with the results of Strahle et al.[1] for finite Reynolds numbers—results that indicate rapid transitions of temperature and x-wise velocity within a layer adjacent to the body.

A final but important implication from Eqs. (22) relates to the numerical treatment of Eqs. (18c) and (18d). Since we known from Eq. (25) that only A_K is at our disposal to satisfy the two boundary conditions at $\eta \to \infty$, we must consider another parameter to be free. Thus, we select $c_{\varepsilon 10}$ appearing in Eq. (24a) so that two conditions can be satisfied. Since in region I we need only $c_{\varepsilon 2}$, we keep its value constant and equal to $c_{\varepsilon 20}$.

If comparison between predicted and measured variations of skin friction and heat transfer is to be carried out, e.g., as in Smith and Kuethe,[10] we need the following relations:

$$\tau_w = \rho^{2/3}\mu^{1/3}k_\infty^{1/6}a^{4/3}x\hat{f}''(0) \tag{26a}$$

$$q_w = (c_p T_\infty/\sigma)\rho^{2/3}\mu^{1/3}k_\infty^{1/6}a^{1/3}(g_w - 1)\hat{g}'(0) \tag{26b}$$

We thus see that the skin friction and heat transfer are proportional to $k_\infty^{1/6}$, a result calling for comment. Stagnation point heat transfer is found experimentally by Smith and Kuethe[10] to be proportional to the $k_\infty^{1/2}$, but the experimental data in support thereof is widely scattered and may not correspond to a sufficiently high Reynolds number $k_\infty/\nu a$ for the present analysis to apply. Moreover, the calculations of Strahle et al.[1] and Traci and Wilcox[7] give other dependencies.

Region III for Grid Turbulence

We now take up the first of two examples of Region III, namely, grid turbulence approaching the stagnation plane with a constant temperature T_∞. The distribution of the v-velocity component is shown in Fig. 1a and suggests the scales applicable to this region, namely, V_0 and V_0/a. We assume that the bluff body is a circular cylinder of radius R and confine attention to the neighborhood of the dividing streamline. Accordingly, we assume the following forms

$$\bar{u} = ax\bar{f}'(\bar{\eta}) \tag{27a}$$

$$\bar{p} = p_0 - (1/2)\rho V_0^2[p_1(\bar{\eta}) + (ax/2V_0)^2 p_2(\bar{\eta})] \tag{27b}$$

where p_0 is the stagnation pressure. From the continuity equation we find

$$\bar{v} = -V_0\bar{f}(\bar{\eta}) \tag{28}$$

In these equations, the independent variable is

$$\bar{\eta} \equiv (ay/2V_0)$$

When these forms and those assumed for the correlations $\overline{v'^2}$ and $\overline{u'v'}$ in Eqs. (4) are employed, the two momentum equations yield

$$\bar{f}\bar{f}'' - \bar{f}'^2 - \delta G'_{uv} = p_2 \tag{29a}$$

$$\delta(G_{uv} + G'_{vv}) + \bar{f}\bar{f}' = -(1/2)p'_1 \tag{29b}$$

where the small parameter is

$$\bar{\delta} = (k_\infty^{1/2}/V_0) \ll 1$$

If $\bar{\delta} \to 0$, we deal with weak turbulence. In this case we see from Eqs. (29) that to lowest order the mean velocity is given by a potential flow solution. As noted earlier, this result is consistent with the assumption made in rapid distortion theory for this flow.

To lowest order in $\bar{\delta}$ the solution of Eqs. (29) satisfying the applicable boundary conditions as $\bar{\eta} \to \infty$ is

$$\bar{f} = 1 - [1/(1+\bar{\eta})^2] \tag{30a}$$

$$\bar{p} = p_0 - \frac{1}{2}\rho V_0^2 \left\{ 1 - \left[1 - \frac{1}{(1+\bar{\eta})^2} \right] + \left[\frac{2}{(1+\bar{\eta})^6} - \frac{6}{(1+\bar{\eta})^4} \right] \left[\frac{ax}{2V_0} \right]^2 \right\} \tag{30b}$$

Matching of regions II and III involves $\bar{\eta} \to 0$ and $\eta \to \infty$.

With the velocity components known, attention focuses on the equations for the turbulent kinetic energy and viscous dissipation; variables for which region III are made dimensionless according to the same scaling used in region II, namely,

$$k = k_\infty \bar{K}(\bar{\eta}), \qquad \varepsilon = a k_\infty \bar{E}(\eta)$$

The equations for $\bar{K}(\bar{\eta})$ and $\bar{E}(\bar{\eta})$ are found from the standard k-ε equations with the velocity components of Eqs. (27), but if $\bar{\delta} \to 0$, the diffusion terms are negligible. Thus, we have for the lowest order description of the approaching stream the equations

$$-[1-(1+\bar{\eta})^{-2}]\frac{\bar{K}'}{2} = 4c_\mu \frac{\bar{K}^2}{\bar{E}} \frac{1}{(1+\bar{\eta})^6} - \bar{E} \tag{31a}$$

$$-[1-(1+\bar{\eta})^{-2}]\frac{\bar{E}'}{2} = \frac{\bar{E}}{\bar{K}}\left[\bar{c}_{\varepsilon 1} 4c_\mu \frac{\bar{K}^2}{\bar{E}} \frac{1}{(1+\bar{\eta})^6} - \bar{c}_{\varepsilon 2}\bar{E} \right] \tag{31b}$$

where we denote the empirical constants in the dissipation equation as $\bar{c}_{\varepsilon 1}$ and $\bar{c}_{\varepsilon 2}$. Here, we follow Champion and Libby[9] and allow both $\bar{c}_{\varepsilon 1}$ and $\bar{c}_{\varepsilon 2}$ to vary according to

$$\bar{c}_{\varepsilon 1} = \bar{c}_{\varepsilon 1\infty} + (c_{\varepsilon\infty} - \bar{c}_{\varepsilon 1\infty})e^{-\bar{\eta}}$$

$$\bar{c}_{\varepsilon 2} = \bar{c}_{\varepsilon 2\infty} + (c_{\varepsilon\infty} - \bar{c}_{\varepsilon 2\infty})e^{-\bar{\eta}}$$

where $\bar{c}_{\varepsilon 1\infty}$ and $\bar{c}_{\varepsilon 2\infty}$ take on standard values and $c_{\varepsilon\infty}$ is the common value at the outer edge of region II, assigned here the value 2.27. Note that as $\bar{\eta} \to \infty$ the production terms vanish and, thus, the equations reduce to those for decaying grid turbulence, behavior consistent with the vanishing of the rate of strain as $\eta \to \infty$ [cf. Eqs. (27)]. These equations are subject to initial conditions, namely,

$$\bar{K}(0) = 1, \qquad \bar{E}(0) = E_\infty = 2c_\mu^{1/2} \tag{32}$$

and describe the changes in turbulence with distance from the stagnation plane.

Region III for Slot Turbulence

We now take up the flow shown in Fig. 1b; a turbulent stream from a slot impinging on a wall. The procedure used in the previous section applies to this case. Thus, the mean velocity components are

$$\bar{u} = \frac{x}{d} V_0 \bar{f}'(\bar{\eta}) \tag{33a}$$

$$\bar{v} = -V_0 \bar{f}(\bar{\eta}) \tag{33b}$$

where $\bar{\eta} \equiv y/d$. Again, the assumption that the turbulent intensity is small applied to the momentum equations implies that turbulent exchange is negligible and, thus, that the mean velocity and the mean pressure are given by potential flow, i.e., by Euler equations. If we confine attention to the neighborhood of the plane of symmetry, the flow is also irrotational and we find that

$$f = \sin[(\pi/2)\bar{\eta}] \tag{34a}$$

$$\bar{p} = p_0 - (1/2)\rho V_0^2[(\pi^2/4)(x/d)^2 + \bar{f}^2] \tag{34b}$$

In this case, the rate of strain parameter becomes

$$a = (\pi/2)(V_0/d)$$

The equations for the turbulent kinetic energy and the viscous dissipation with the scaling of Eqs. (3) become

$$-\frac{2}{\pi}\sin\left(\frac{\pi}{2}\bar{\eta}\right)\bar{K}' = 4c_\mu \frac{\bar{K}^2}{\bar{E}}\cos^2\left(\frac{\pi}{2}\bar{\eta}\right) - \bar{E} \tag{35a}$$

$$-\frac{2}{\pi}\sin\left(\frac{\pi}{2}\bar{\eta}\right)\bar{E}' = \frac{\bar{E}}{\bar{K}}\left[4c_{\varepsilon 1} c_\mu \frac{\bar{K}^2}{\bar{E}}\cos^2\left(\frac{\pi}{2}\bar{\eta}\right) - c_{\varepsilon 2}\bar{E} \right] \tag{35b}$$

These equations are to be solved subject to the initial conditions

$$\bar{K}(0) = 1, \qquad \bar{E}(0) = 2c_\mu^{1/2}$$

The requirement that the solutions match with those for region II as $\bar{\eta} \to 0$ necessitates that at $\bar{\eta} = 0$ $c_{\varepsilon 1} = c_{\varepsilon 2} = c_\varepsilon$. Thus, in this case, we require that these two constants take on their standard values at the exit of the slot, i.e., $c_{\varepsilon 1\infty}$ and $c_{\varepsilon 2\infty}$. Accordingly, we take

$$c_{\varepsilon 1} = c_\varepsilon - r(c_\varepsilon - c_{\varepsilon 1\infty})(1 - e^{-\bar{\eta}}), \quad c_{\varepsilon 2} = c_\varepsilon - r(c_\varepsilon - c_{\varepsilon 2\infty})(1 - e^{-\bar{\eta}})$$

where

$$r \equiv 1/(1 - e^{-1})$$

III. Numerical Results

We summarize here the values of the many empirical constants arising in this analysis. We use:

$$\sigma_k = 1, \qquad \sigma_\varepsilon = 1.3, \qquad \sigma_T = 0.9$$

$$c_\mu = 0.09$$

$$\bar{c}_{\varepsilon 1\infty} = 1.44, \qquad \bar{c}_{\varepsilon 2\infty} = 1.92, \qquad c_{\varepsilon\infty} = 2.27$$

$$A_1 = 0.05, \qquad A_\mu = 0.0165, \qquad A_l = 20.5$$

From Eq. (23) we find $c_{\varepsilon 20} = 0.256$. The molecular Prandtl number is taken to be $\sigma = 0.7$.

We first consider the solutions for region II since they provide the values needed for the treatment of region I. In Fig. 2, we show these solutions, which involve determination of $A_K = 3.20$ and $c_{\varepsilon 10} = 1.30$. From Eqs. (24) we have $A_E = 2.10$. Although $K(\eta)$ monotonically approaches unity, the viscous dissipation, as reflected in $E(\eta)$, involves a slight overshoot before approaching its asymptotic value. Note that the edge of region II corresponds roughly to $\eta = 2$, implying that the thickness of the shear layer is approximately $2k_\infty^{1/2}/a$.

Determination of the value of $\hat{\eta}$ at which the conditions applicable to the outer edge of region I are imposed calls for several considerations. First, these conditions require that the two empirical functions f_μ, f_2 assume values close to unity. The size of the empirical constant A_μ appearing in the first of these functions determines the range of $\hat{\eta}$ on the basis of this requirement; if we demand that $f_\mu \approx 0.98$, we find that the

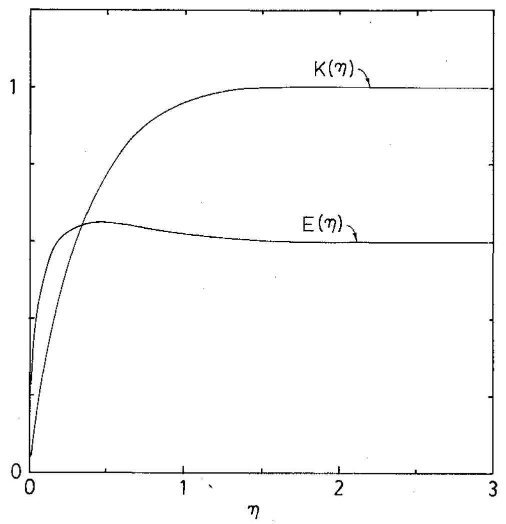


Fig. 2 Solutions for region II.

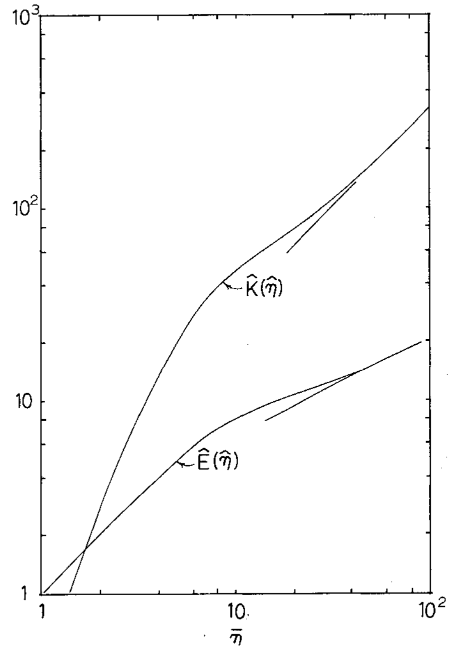
numerical integration must extend to $\hat{\eta} = 26$, an unexpectedly large value. However, a further requirement for this range relates to the negligibility of molecular compared with turbulent transport at the outer edge of region I, a requirement calling for the $c_\mu \hat{K}^2/\hat{E}$ terms in Eqs. (15b) and (15c) to be large compared with unity. Examination indicates that the $\hat{E}$ equation makes the most stringent demands in this regard; with the values of the empirical constants and of A_K cited earlier the turbulent transport term approaches $0.354\hat{\eta}$. Thus, if this term is 20 times greater than the molecular term, we find that the integration must be extended to $\hat{\eta} = 60$, whereas, if the turbulent contribution is to be a hundred times greater than the molecular, the boundary conditions must be imposed at $\hat{\eta} \approx 300$. This finding indicates that approximate but analytic solutions to Eqs. (15b) and (15c) reflecting the asymptotic approach of $\hat{K}(\hat{\eta})$ and $\hat{E}(\hat{\eta})$ to $A_K\hat{\eta}$ and $A_E\hat{\eta}^{1/2}$, respectively, are called for. Such a solution for $\hat{E}(\hat{\eta})$ indicating an algebraic approach to its asymptotic form can be rather simply obtained, whereas that for $\hat{K}(\hat{\eta})$ is obtainable but cluttered. Even with such solutions we find it necessary to carry out the integration to $\hat{\eta} = 60$.

In Fig. 3, we present the solutions for region I; solutions involving determination of $\hat{E}(0) = 2\alpha_K = 1.07$, $\hat{f}''(0) = 0.157$, and $\hat{g}'(0) = 0.146$. Figure 3a shows the distributions of the scaled turbulent kinetic energy and viscous dissipation [cf. Eqs. (14)] in terms of $\hat{K}(\hat{\eta})$ and $\hat{E}(\hat{\eta})$, whereas Fig. 3b shows the velocity and temperature distributions in terms of $\hat{f}'(\hat{\eta})$ and $\hat{g}(\hat{\eta})$. The different dependence of $\hat{K}(\hat{\eta})$ and $\hat{E}(\hat{\eta})$ on $\hat{\eta}$ at the two edges of region I are clearly seen in Fig. 3a. The velocity and temperature profiles differ only slightly since the ratio σ/σ_T in Eq. (17) is close to unity.

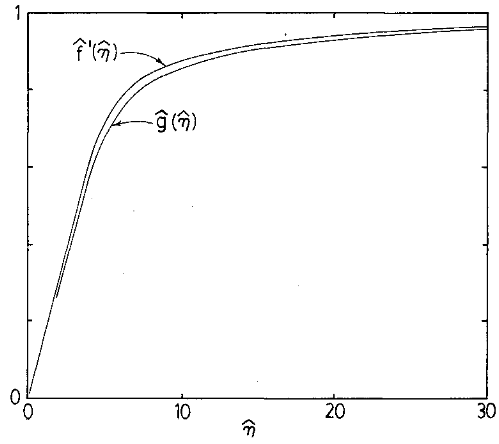
If we identify the edge of region I with $\hat{\eta} = 60$, the ratio of the sublayer and shear layer thicknesses is readily found to be

$$\frac{\delta_{vsl}}{\delta_{sl}} = 30\,\hat{\delta}^{2/3}$$

This result has significant implications regarding the applicability of the present analysis. If the viscous sublayer is to be no more than 10% of the shear layer in thickness, this equation implies that $\hat{\delta}$ must be on the order of 10^{-4} or less. From our earlier considerations of the size of $\hat{\delta}$, we find that this requires for grid turbulence the Reynolds number $V_0 R/\nu$ to be of order 10^6 even if $k_\infty^{1/2}/V_0 = \mathcal{O}(10^{-1})$, a relatively intense turbulence for grid flows. Since, in most experiments, relating to the influence of freestream turbulence on stagnation point skin friction and heat transfer these values are at best marginally realized, we conclude that it is likely that any disagreement between the present analysis and existing experimental results is due to the value of $\hat{\delta}$ associated with



a) Scaled turbulent kinetic energy and viscous dissipation.



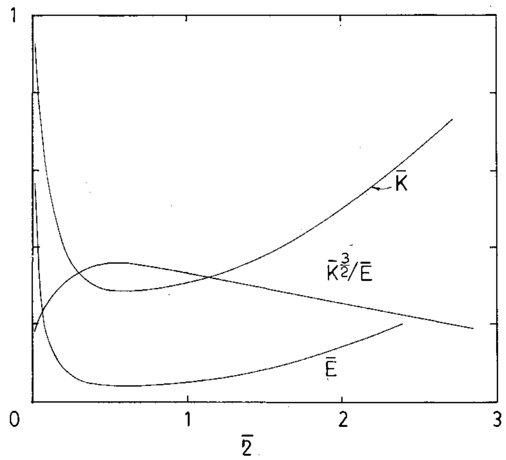
b) The x-wise velocity component and the mean temperature.

Fig. 3 Solutions for region I.

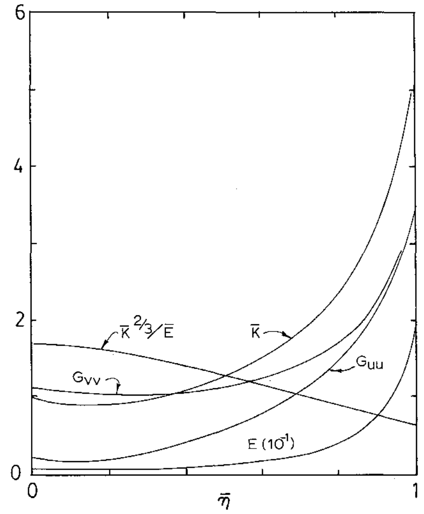
experimental conditions being insufficiently small. Similar conclusions relate to the applicability of the present analysis to slot turbulence. Despite this important result, the present asymptotic analysis has the virtue of clearly establishing the nature of the viscous sublayer and shear layer in stagnating turbulence as the Reynolds number $k_\infty/\nu a$ increases. In particular, we conclude that the viscous sublayer in these flows tends to be relatively thick compared with its counterpart in turbulent boundary layers, a conclusion consistent with the entire change in the mean velocity and mean temperature occurring within the viscous sublayer.

The solutions for region III in the case of grid turbulence are available from Ref. 9 but are repeated here as Fig. 4a for completeness. Also shown is the variation of a measure of a turbulent length scale, namely, $\bar{K}^{3/2}/\bar{E}$. From this figure, we see that in the approaching stream the turbulence decays and the turbulent length scale grows until the influence of the rate of strain field associated with the body comes into play in the neighborhood of $\bar{\eta} \approx 0.5$.

In Fig. 4b, we show the corresponding results for region III in slot turbulence with $c_\varepsilon = 2.27$, the value established by comparison with experimental data in Ref. 9. However, we note that the results are sensitive to the value of this coefficient and comparison with experimental data may suggest



a) Grid turbulence (from Ref. 9)



b) Slot turbulence

Fig. 4 Solutions for region III.

a different value for slot turbulence. Qualitatively, the results in Fig. 4b agree with those for grid turbulence in Fig. 4a but differences in the rate of strain fields lead to a more rapid decrease in turbulent kinetic energy as the stagnation plane is approached.

IV. Concluding Remarks

We develop here a consistent k-ε calculation for all three regimes of a stagnating turbulent flow in the limit of a large Reynolds number $k_\infty/\nu a$ where k_∞ and a are the turbulent kinetic energy in the stream just external to the shear layer and the mean rate of strain, respectively. To achieve this consistency, it is necessary to adjust the empirical constants in the dissipation equation. Close to the body, their values are determined by matching requirements between regions I and II and between regions II and III. The analysis of the viscous sublayer and the shear layer applies to any case of stagnating turbulence. Here, two examples are discussed: decaying grid turbulence encountering a bluff cylindrical body and turbulence from a slot impinging on a wall.

The analysis shows that the viscous sublayer and the shear layer differ significantly from their counterparts in turbulent boundary layers as a consequence of the differences in the mean rates of strain. For example, the viscous sublayer is found to involve a balance between diffusion and dissipation, to account for the entire change in streamwise velocity and temperature from the wall to freestream values and to be relatively thick. Comparison of the relative thickness of the viscous sublayer and the shear layer suggests that this analysis applies only for exceedingly small values of the expansion parameter, i.e., for large Reynolds numbers and intense turbulence. Thus, any discrepancy between this analysis and experiment may be due to an inappropriately large value for the expansion parameter $\hat{\delta}$. Despite this limitation, the analysis clearly exposes the character of viscous sublayers and shear layers in stagnating turbulence.

It would appear to be of interest to apply Reynolds stress closure to the two cases of stagnating turbulence treated here. A start to this end is provided by Taulbee and Tran[8] in their treatment of the approaching stream, our region III, for grid turbulence. However, it is easy to establish that the modifications of the constants in the viscous dissipation equation required to match the solutions for regions II and III called for in the present k-ε analysis are also required in Reynolds stress modeling.

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